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Trajectories of high-energy cosmic rays in the galactic disk

K. O. THIELHEIM and W. LANGHOFF
Institut für Reine and Angewandte Kernphysik, Universität Kiel, Kiel, West Germany

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Abstract. Trajectories of high-energy cosmic ray particles, the rigidity of which corresponds to protons of energy $E_0 = 10^{18}$ ev, have been calculated numerically, using a quasi-longitudinal model of the magnetic field in the galactic disk. In this way an idea of cosmic radiation transfer at extremely high energies is obtained under specified assumptions concerning the magnetic field in the galactic disk. The galactic or extragalactic origin of particles reaching the Earth, as well as the angular distribution of such particles, is discussed. The possibility of particle transfer from the galactic centre to the Earth is investigated.

1. Introduction

The transfer of high-energy cosmic ray particles through interstellar space is subject to interactions with rest gas, 3 °K black-body radiation and magnetic fields. These interactions are expected to influence the combined distribution of energy, mass (or charge) and direction of cosmic ray particles, which is investigated in extensive air shower experiments. The object of this paper is the interaction of extremely high-energy cosmic ray particles with interstellar magnetic fields. Therefore, we are concerned with two problems: (*a*) the formulation of a model of the magnetic field of the galactic disk and (*b*) the theoretical treatment of high-energy particle transfer.

(*a*) A great amount of empirical information on the magnetic field of the galactic disk has become available in the last few years and we find it useful to present a short review of methods and results in § 2 before discussing the model in § 3 of this paper. This is the quasi-longitudinal version of a model, which has been constructed as a basis for calculations on cosmic ray transfer (Thielheim 1968, unpublished), and is intended to be a synthesis of empirical data and theoretical arguments on the large-scale ordered contributions to the magnetic field of the galactic disk. Therefore, the model is supposed to have some essential structural features in common with the existing field, but is not to be taken as a map of the latter field. The model does not comprise details of the local structure (e.g. bending, branching or divergence), of the central structure (which due to the smallness of the central region is of no interest here) or of the halo structure (of which we have too little information).

(*b*) At extremely high magnetic rigidities of cosmic ray particles, corresponding to proton energies greater than about 10^{17} ev, the large-scale ordered contribution to the magnetic field of the galactic disk is considered to be of main importance for particle transfer. The calculation of individual particle trajectories seems to be the appropriate method to obtain an idea of particle transfer in this energy region (Thielheim and Langhoff 1968 a). Thus, our problem is similar to Störmer's work on the polar aurora and on solar protons (Störmer 1930, 1934 a, 1934 b, 1936 a, 1936 b, 1950, Vallarta 1933, 1935 a, 1935 b, 1939 a, 1939 b, 1948, 1961, Vallarta *et al.* 1939, Johnson 1938, Lemaitre and Vallarta 1933, 1936 a, 1936 b, Schlüter 1951, Lüst *et al.* 1955, Quenby and Webber 1959, Quenby and Wenk 1962, McCracken *et al.* 1962, Smart and Shea 1967). Calculations on the structure of the galacto-magnetic cut-off result in a possible explanation of the 'knees' of the primary energy spectrum (Thielheim and Langhoff 1968 b). But details of the Störmer cones and of the angular distribution are expected to be influenced by irregularities of the field, since the region of transition from galactic to extragalactic contributions extends to the range of magnetic rigidity corresponding to protons of energy about 10^{16} ev, where such fluctuations will be important. Life time of cosmic ray particles within the region of the galaxies as a function of magnetic rigidity is obtained from systematic calculations of trajectories in the region of high rigidity. In the range of rigidity corresponding to protons of energy below about 10^{16} ev, stochastic methods of particle transfer will be applicable (Unsöld 1951, Middendorf 1968, private communication), starting from specified assumptions on the spectrum of irregularities.

2. A short review of empirical methods and results on the magnetic field of the galactic disk

Empirical data on the magnetic field of the galactic disk have been obtained by a great number of authors through application of the following methods:

(i) Measurement of the polarization of starlight. The polarization is assumed to be due to the Davis–Greenstein mechanism, i.e. the reflection by elongated paramagnetic interstellar dust particles rotating with their short axis of inertia aligned parallel to the lines of force (Davis and Greenstein 1951). The orientation of the electric vector is parallel to the projection of the lines of force. The degree of polarization is typically of the order of 10%. The direction of the projection of the interstellar magnetic field irrespective of its sign may be inferred from such observations. Simultaneously, an estimation of the distance may be obtained considering the extinction of the blue part of the spectrum. This method is restricted to a range of distance of a few thousand parsec from the Sun. Polarization surveys including data of thousands of stars are available (Hall 1958, Behr 1959, Serkowski 1962, Appenzeller 1966 and others).

(ii) Measurement of the brightness and polarization of synchrotron radiation emitted by electrons in the GeV range (Biermann and Davis 1960, Ginzburg and Syrowatzki 1964, 1965, Mathewson and Milne 1964, Gardner and Whiteoak 1966). The energy spectrum of interstellar electrons is assumed to be of the potential form. Absolute values of the density of electrons and of the exponent of the energy spectrum are inferred from measurements in the neighbourhood of the Earth. The intensity of synchrotron radiation is obtained in terms of the projection of magnetic field strength and of the energy spectrum of electrons, $I_\nu \propto H_\perp{}^{(\gamma+1)/2}\nu^{(1-\gamma)/2}$. The orientation of the electric vector is perpendicular to the projection of the lines of force. The degree of polarization optimally emitted by an isotropic distribution of electrons with exponent $\gamma = 2{\cdot}4$ of the energy spectrum in a homogeneous magnetic field is 72%. The observed degree of polarization is typically of the order of several per cent only, owing to contributions from regions with different projections of the field strength and to modifications by regions of different Faraday thickness between source and observer. This method is applicable in a range of distance of a few kpc from the Sun. Polarization surveys are in general agreement with optical data.

(iii) Measurement of the Faraday rotation of the plane of the electric vector around the line of sight of linearly polarized radio-frequency radiation from extragalactic sources (Cooper and Price 1962, Gardener and Whiteoak 1963, Morris and Berge 1964, Gardener andDavies 1966, Berge and Seielstad 1967). The angle of rotation is proportional to the square of the wavelength. The constant of proportionality is the Faraday rotation measure $R = 8{\cdot}1\times10^5\int N_e H_\parallel \, dL$, where N_e is electron density (in cm^{-3}), on which assumptions must be made, $H_\parallel$ is the line of sight component of the magnetic field (in gauss) and dL is the element of the line of sight (in pc). Positive values of R correspond to a field directed towards the observer. Values of R are typically of the order of 10–100 (rad m^{-2}).

(iv) Zeeman splitting of the 21-cm line of absorption by neutral hydrogen (Davies *et al.* 1962, Bolton and Wild 1957). The line of sight component of the magnetic field gives rise to the splitting into two contributions, which are left-hand and right-hand polarized respectively. The separation is 28 $second^{-1}/10^{-5}$ G, which is extremely small. Therefore, the best we can expect from such measurements for the moment are upper limits of the field strength in rather dense clouds, from the observation of Cassiopeia A for example.

(v) There are further possibilities for rather indirect estimations on the structure of the interstellar field, e.g. observations of the structure of filament nebula (Shain 1955), which are suspected to be aligned along the lines of force, although there is the difficulty of distinguishing them from structures caused by shock fronts for example.

Indirect conclusions on the structure of the magnetic field in the galactic disk may also be derived from theoretical arguments:

(i) Stability of the galactic arms (Chandrasekhar and Fermi 1953), which in a first approximation may be understood as a consequence of the equilibrium between self-gravitation and magnetic forces.

(ii) Dynamical theories of the galactic arms have to consider stability in the presence of differential rotation (Prendergast and Burbidge 1960, Lindblad 1960, 1964, Wentzel 1960, Toomre 1964, Oki *et al.* 1964, Lyttleton and Bondi 1964, Goldreich and Lydon-Bell 1965). A still non-existent theory of the genetics and development of spiral galaxies will have to include magnetic forces as well as gas-dynamic and gravitative forces (Piddington 1964, 1966, 1967). Estimations may be based on the application of the virial theorem (Biermann and Davis 1960).

(iii) Morphological comparison of our own galaxy with other galaxies of the middle Sb type under the assumption of correlations between the distributions of young stars, gas and magnetic field. Concluding this short review, we want to refer to some data, which have been obtained by various authors by application of the aforementioned methods:

(i) There is only a little information available on the magnetic field of the galactic centre (Westerhout *et al.* 1958, Okuda and Tanaka 1967). Since there is no need to include the structure of the magnetic field of the galactic centre in the model presented below, we will not discuss this point further.

(ii) Very little is known also of the magnetic field of the halo. Biermann and Davis (1960) estimated $\langle H_{\mathrm{H}} \rangle \lesssim 5 \times 10^{-6}$ G assuming $\gamma = 7/3$ for the exponent of the energy spectrum of electrons, similarly Okuda and Tanaka (1967) found $\langle H_{\mathrm{H}} \rangle \leqslant 3 \times 10^{-6}$ G with $\gamma = 2{\cdot}5$. The existence of a large-scale ordered field of considerable strength in the halo is not well established. Unsöld (1967) argues that there is no field of this kind. Sciama considers the halo field in connection with the metagalactic field (Sciama 1962). Therefore, we have not tried to include the halo field in the model presented in the next section.

(iii) Many more data are available on the magnetic field of the galactic disk. The mean value found by Biermann and Davis (1960) is $\langle H_{\mathrm{D}} \rangle = 2 \times 10^{-5}$ G. Okuda and Tanaka (1967) found $\langle H_{\mathrm{D}} \rangle = 6 \times 10^{-6} - 1{\cdot}2 \times 10^{-5}$ G. This is quite in agreement with measurements of the Faraday rotation measure leading to $\langle H_{\mathrm{D}} \rangle = 10^{-5 \pm 1}$ G (Woltjer 1966) and with $\langle H_{\mathrm{D}} \rangle \simeq 9 \times 10^{-6}$ G as given by Sironi (1965). A mean value of about 10^{-5} is considered to be reasonable (van der Hulst 1967) in our model. According to the observation of a belt of maximal optical and radio-frequency polarization along a great circle, which is about 60° wide and cuts the galactic equator at about $l^{\mathrm{II}} = 320°–20°$ and $l^{\mathrm{II}} = 120°–180°$ (Mathewson 1964, 1965), the direction of the local field is about $l^{\mathrm{II}} = 70°$, $b^{\mathrm{II}} = 0°$. The observation that the direction of the local field (and of the local stellar arm as well as the gas arm) differs from about $l^{\mathrm{II}} = 90°$ may be explained by bending, branching or divergence (Sharpless 1965, and Bingham and Shakeshaft 1967). These details have not been included in the model presented below.

From the observation of positive Faraday measures for $\sim 340° < l^{\mathrm{II}} < \sim 160°$, $b^{\mathrm{II}} > 10°$, and of negative Faraday measures for $\sim 160° < l^{\mathrm{II}} < \sim 340°, b^{\mathrm{II}} < 10°$ (Morris and Berge 1964, Gardner and Davis 1966), the direction of the local field is found to be opposite on either side of a plane which is parallel to the stellar galactic equator at a northern distance of about 100 pc.

These features of the local field structure have been generalized by Berge and Seielstad (1964) in terms of the quasi-longitudinal model of the magnetic field of the galactic arm, which has also been adopted in the model presented below. The quasi-longitudinal model may be understood as a refinement of the longitudinal model proposed by Chandrasekhar and Fermi (1953). An alternative structure has been proposed in terms of the helical model, in which the lines of force have the form of helices winding around the spiral arm (Behr 1959, Hoyle and Ireland 1961, Ireland 1961, Stepien 1964, Hornby 1966).

3. A model for the large-scale magnetic field of the galactic disk

The model is formulated with the help of cylindrical coordinates R, z, ϕ. $z = 0$ is the 'galactomagnetic' equatorial plane, where, according to the quasi-longitudinal model, a reversal of the direction of the field within the galactic arms takes place. The geometrical form of the galactic arms is given with the help of a spiral function

$$\phi(R) = \frac{b}{k} R \arctan \frac{R}{k} + \phi_0. \tag{1}$$

There are two arms starting from opposite sides of the galactic centre. The windings become equidistant at larger distances from the centre. ϕ_0 may be taken equal to zero. The parameters are $b = 1$ and $k = 1{\cdot}5$ kpc. If the position of the Sun is given by $R_\odot = 10$ kpc, $\phi_\odot = 6{\cdot}5°$ and $z_\odot = -85$ pc, the distances of the inner windings for this value of ϕ are 4 kpc, 7 kpc and 10 kpc (figure 1).

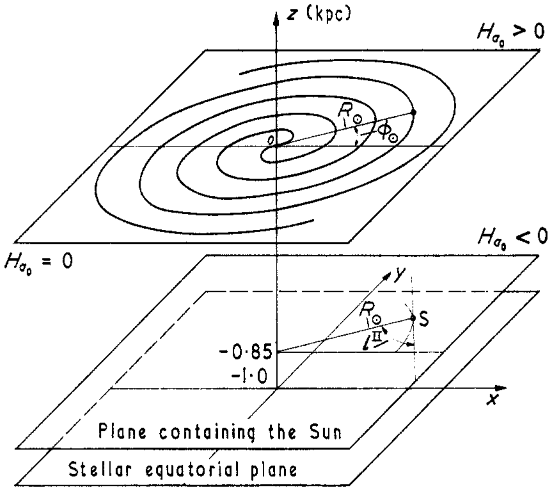

Figure 1. Illustration of 'galactomagnetic coordinates' R, z and ϕ. $z = 0$ is the 'galactomagnetic equatorial plane', where—according to the quasi-longitudinal model—H_{a_0} changes sign. $z = -85$ pc is the plane containing the Sun.

The magnetic field is given with the help of a unit vector

$$\boldsymbol{a}_0 = \boldsymbol{R}_0 \cos \epsilon + [\boldsymbol{z}_0, \boldsymbol{R}_0] \sin \epsilon \tag{2}$$

where the angle ϵ is a function of R alone

$$\epsilon = \arctan \left(\frac{b}{k} R \arctan \frac{R}{k} + \frac{bR^2}{k^2 + R^2} \right). \tag{3}$$

This vector is obtained by rotation of the tangent vector to the spiral curve around the galactic centre.

In the quasi-longitudinal version of the model (Thielheim 1968, unpublished), the magnetic field strength is proposed to be of the form

$$\boldsymbol{H}_\mathrm{D} = \boldsymbol{a}_0 H_{a_0} + \boldsymbol{z}_0 H_{z_0} \tag{4}$$

with

$$H_{a_0} = cz \exp\left(-\frac{z^2}{z_0^2}\right) \exp\left(-\frac{R^2}{R_1^2}\right)\left\{1 - \exp\left(-\frac{R^2}{R_2^2}\right)\right\} \times [1 + a^2 \cos^2 \{\phi - \phi(R)\}].$$

According to the definition of $\boldsymbol{a}_0$, this is the component of the field in a direction approximately parallel (or antiparallel) to the galactic arms. Parameters are $z_0 = 0{\cdot}175$ kpc,

$R_1 = 10$ kpc, $R_2 = 2$ kpc. Thus the field vanishes rapidly outside the region of the galactic disk. The form of decrease has been chosen for practical reasons and certainly may not be very realistic. The ratio of field strength within the arms to field strength in between the arms is adjusted by parameter a, which has been given the value $a = 2$. As is obvious from the last factor of (5), the field is of considerable strength only within the galactic arms. The mean value of the field strength in the galactic disk may be adjusted by giving an appropriate value to parameter c, which we have chosen to be $c = 5 \times 10^{-5}$ G kpc^{-1}. The form of H_{a_0} is illustrated by figures 2 and 3, which show lines H_{a_0} = const. in the planes

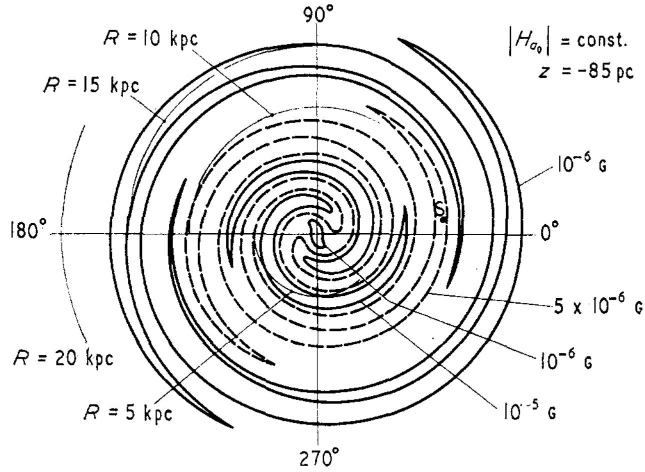

Figure 2. Lines H_{a_0} = const. in the plane $z = -85$ pc for the quasi-longitudinal version of the model.

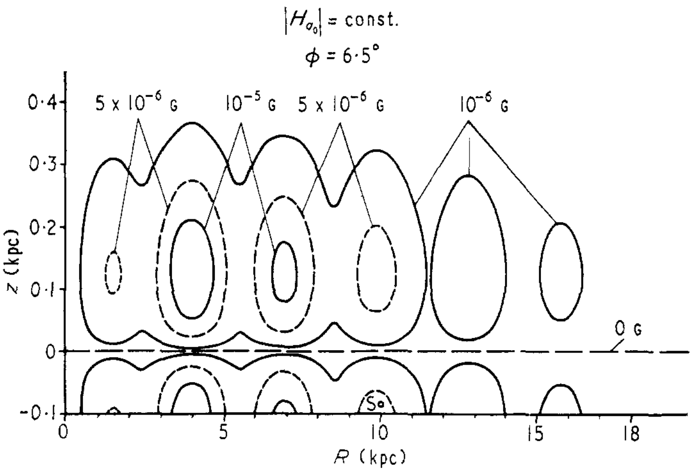

Figure 3. Lines H_{a_0} = const. in the plane $\phi = 6{\cdot}5°$ for the quasi-longitudinal version of the model.

$z = -85$ kpc and $\phi = 6{\cdot}5°$ respectively, which contain the position of the Sun. H_{z_0} is obtained from (5) by means of div $\boldsymbol{H}_{\mathrm{D}} = 0$ and appropriate boundary conditions, as is demonstrated by figures 4 and 5.

4. Results and conclusions

Since we are interested in trajectories hitting the Earth, we have formally calculated trajectories of antiparticles starting from the Earth in various directions, which are indicated in figure 6. All trajectories shown in this paper are for magnetic rigidity corresponding to

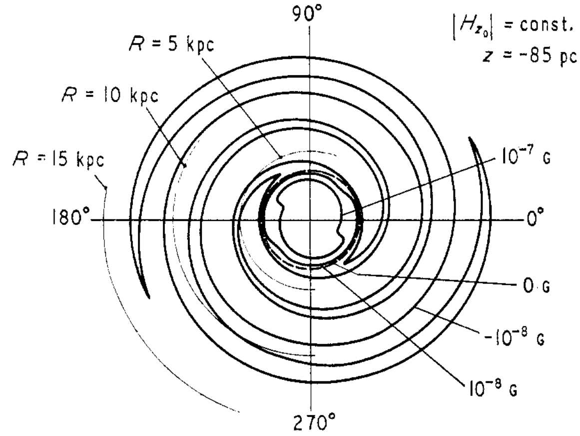

Figure 4. H_{z_0} = const. in the plane $z = -85$ pc for the quasi-longitudinal version of the model.

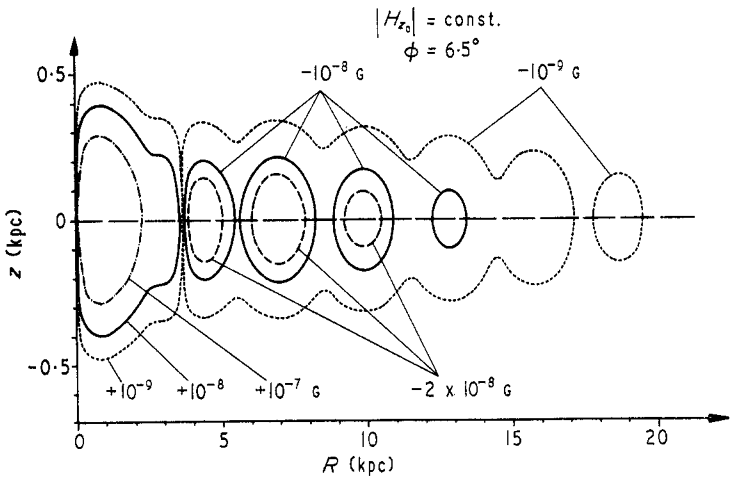

Figure 5. Lines H_{z_0} = const. in the plane $\phi = 6{\cdot}5°$ for the quasi-longitudinal version of the model.

protons of energy 10^{18} ev. Projections on the equatorial plane are shown in figure 7. From this picuture one may draw the following conclusions:

(i) There is no transfer of protons of energy 10^{18} ev, or of other particles of equal magnetic rigidity, from the galactic centre to the Earth (with the possible exception of re-entrant trajectories) within the frame of this model.

(ii) There is a kind of focusing effect of the magnetic field, since almost all trajectories cut the sphere of radius 20 kpc around the galactic centre on one hemisphere. This may be interesting if one is discussing anisotropic incidence of extragalactic particles on the galactic disk.

(iii) Only a small fraction of the region of the galactic disk is covered by the trajectories. The sources of these particles should be either in this region or outside the disk. This could be interpreted in favour of extragalactic (or metagalactic) origin of such particles.

(iv) There is no forbidden region of solid angle for extragalactic particles of this rigidity hitting the earth. Thus, by application of Liouville's theorem (Fermi and Rossi 1963, Swann 1933) one may—under the assumption of isotropic incidence of extragalactic particles on the galactic disk—conclude that extragalactic protons of energy 10^{18} ev are isotropic in the neighbourhood of the Sun.

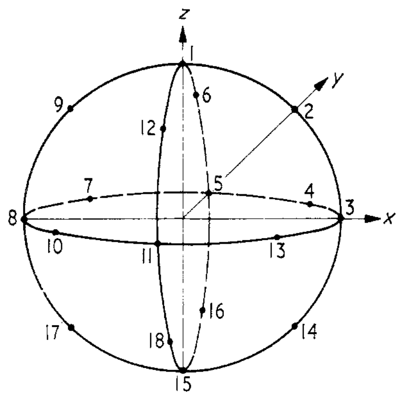

Figure 6 Identification of trajectories hitting the Earth from various directions.

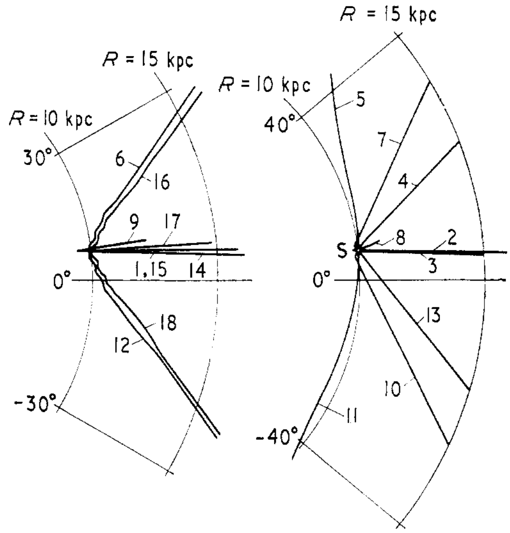

Figure 7. Projection of trajectories 1–18 of particles with magnetic rigidity corresponding to protons of energy 10^{18} ev on the (R, ϕ) plane, which is identical with the (x, y) plane.

Other projections of particle trajectories are shown for illustration in figures 8, 9 and 10.

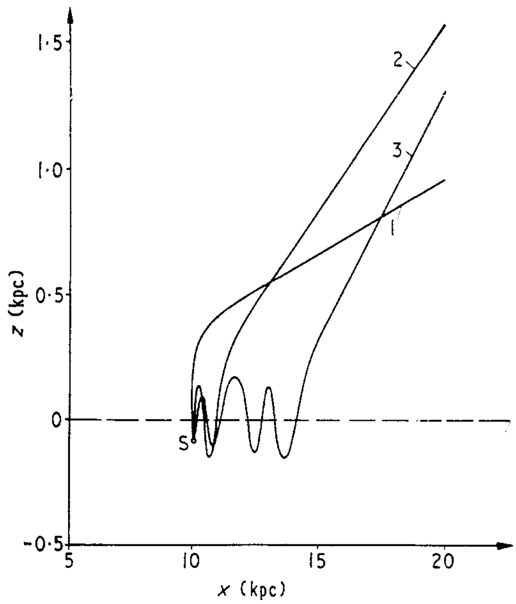

Figure 8. Projection of trajectories 1–3 of particles with magnetic rigidity corresponding to protons of energy 10^{18} ev on the (x, z) plane, which is identical with the plane $\phi = 0$.

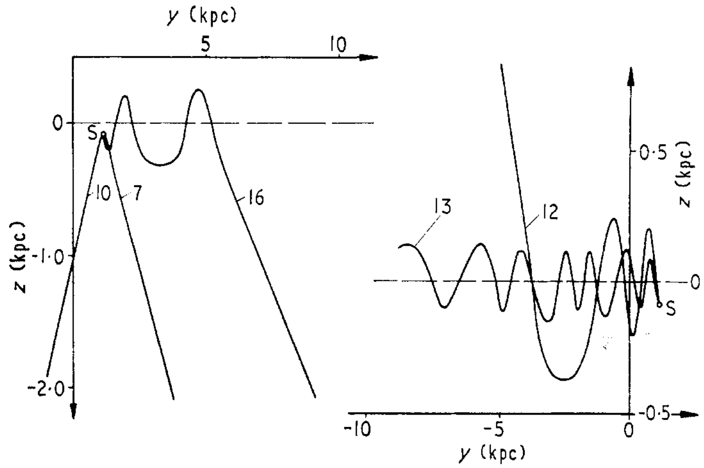

Figure 9. Projection of trajectories 7, 10, 12, 13 and 16 of particles with magnetic rigidity corresponding to protons of energy 10^{18} ev on the (y, z) plane, which is identical with the plane $\phi = 90°$.

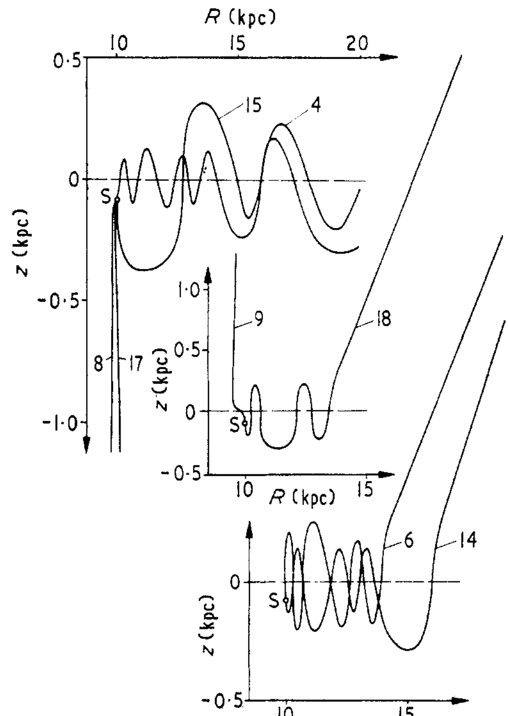

Figure 10. Projection of trajectories 4, 6, 8, 9, 14, 15, 17 and 18 of particles with magnetic rigidity corresponding to protons of energy 10^{18} ev on the (R, z) plane.

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